LBNL–47??1
UTPT–01–05
CALT–68–2316
hep-ph/0103020

# Comment on studying the corrections to factorization in $B \to D^{(*)}X$

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## Abstract

We propose studying the mechanism of factorization in exclusive decays of the form $B \to D^{(*)}X$ by examining the differential decay rate as a function of the invariant mass of the light hadronic state $X$. If factorization works primarily due to the large $N_c$ limit then its accuracy is not expected to decrease as the $X$ invariant mass increases. However, if factorization is mostly a consequence of perturbative QCD then the corrections should grow with the $X$ invariant mass. Combining data for hadronic tau decays and semileptonic $B$ decays allows tests of factorization to be made for a variety of final states. We discuss the examples of $B \to D^*\pi^+\pi^-\pi^-\pi^0$ and $B \to D^*\omega\pi^-$. The mode $B \to D^*\omega\pi^-$ will allow a precision study of the dependence of the corrections to factorization on the invariant mass of the light hadronic state.

An understanding of nonleptonic $B$ decays is important for the study of $CP$ violation at $B$ factories. However, there is at present no systematic, model-independent treatment of all such decays. In certain kinematic situations, factorization, according to which the matrix element of four-quark operators can be written as the product of pairs of matrix elements of two-quark operators, can be justified using perturbative QCD [1,2]. An example is $B \to D^{(*)}X$, where $X$ denotes a hadronic state with low invariant mass, and the $D^{(*)}$ inherits the light degrees of freedom from the $B$ meson. The large $N_c$ limit of QCD [3] provides a different explanation of factorization, which is independent of the kinematics of the final state. Factorization has been shown to be consistent with experiment in certain two-body decays, such as $B \to D^{(*)}\pi$ and $D^{(*)}\rho$. Currently, this represents a test of factorization at the 10–20% level [4]. Factorization has also been observed to hold within the (presently sizable) errors in decays of the type $B \to D^{(*)} D_s^{(*)}$, where no perturbative QCD justification has been presented [5].

In this paper we examine how the decays $B \to D^{(*)}X$, where $X$ is a final state containing two or more light hadrons, allow us to study the corrections to factorization[1], and help determine the role that perturbative QCD plays. In such decays, factorization can be studied as a function of $m_X$, the invariant mass of the final hadronic state produced by the light quark current. The perturbative QCD arguments for factorization depend on the light quarks being produced in an almost collinear state (for this reason, it has also been argued that factorization should hold for $B \to D^{(*)} + \text{jet}$ [6]). Corrections are therefore expected to grow with $m_X/E_X$, where $E_X$ is the energy of the hadronic final state in the $B$ rest frame, since this quantity characterizes the deviation of the hadronic state $X$ from the light cone. However, if factorization works primarily due to the large $N_c$ limit then its accuracy is not expected to decrease as $m_X$ increases. The behavior of factorization violations with $m_X$ can therefore distinguish the perturbative QCD explanation of factorization from that of the large $N_c$ limit. Naïvely, the $1/N_c^2$ terms that violate factorization are $O(10\%)$ in the amplitude and $O(20\%)$ in the rate. When factorization is tested with a greater precision than this, it may be possible to see corrections to factorization which grow with $m_X$, which would provide evidence that perturbative QCD plays an important role in factorization.

Nonleptonic $B$ meson decays arise dominantly due to the weak Hamiltonian

$$H^{\rm nl} = \frac{4G_F}{\sqrt{2}}\, V_{cb} V_{uq}^* \Big[ c_1(m_b)\, (\bar c\, \gamma_\mu P_L\, b)\, (\bar q\, \gamma^\mu P_L\, u) + c_2(m_b)\, (\bar q\, \gamma_\mu P_L\, b)\, (\bar c\, \gamma^\mu P_L\, u) \Big] \,, \qquad (1)$$

where $P_L = (1-\gamma_5)/2$ and $q = d, s$. The Hamiltonian in Eq. (1) is renormalized at the scale $\mu = m_b$. The coefficients $c_i$ are given by $c_{1,2} = (c_+ \pm c_-)/2$, where

$$c_+(\mu) = \left[ \frac{\alpha_s(\mu)}{\alpha_s(M_W)} \right]^{-6/23} , \qquad c_-(\mu) = \left[ \frac{\alpha_s(\mu)}{\alpha_s(M_W)} \right]^{12/23} , \qquad (2)$$

in the leading logarithmic approximation. At next-to-leading order, $c_1(m_b) = 1.13$ and $c_2(m_b) = -0.29$ for $\alpha_s(M_Z) = 0.118$. Under the factorization hypothesis, the hadronic matrix element of $H^{\rm nl}$ can be written as

[1]These decays have also been discussed as tests of factorization by Reader and Isgur [7].

$$\langle X D^{(*)} | H^{\rm nl} | B \rangle = \frac{4G_F}{\sqrt{2}}\, V_{cb} V_{uq}^* \left( c_1(m_b) + \frac{1}{3}\, c_2(m_b) \right) \langle D^{(*)} | \bar{c} \gamma_\mu P_L b | B \rangle \langle X | \bar{q} \gamma^\mu P_L u | 0 \rangle + \dots \,, \tag{3}$$

where the ellipses denote factorization violating terms. The first matrix element can be determined from semileptonic $B \to D^{(*)} \ell \bar\nu$ decays, while the second can be determined from the hadronic tau decay $\tau \to X \nu_\tau$.

The differential rate for tau decay to a given hadronic final state $X$ may be written as

$$\frac{{\rm d}\Gamma(\tau \to X \nu_\tau)}{{\rm d}m_X^2} = \frac{G_F^2 |V_{ud}|^2}{32\pi^2\, m_\tau^3}\, (m_\tau^2 - m_X^2)^2\, (m_\tau^2 + 2m_X^2)\, v_X(m_X^2) \,, \tag{4}$$

where $m_X^2$ is the invariant mass-squared of the hadronic final state $X$, and $v_X(m_X^2)$ is related to the transverse part of the vacuum polarization amplitude. This holds in general even if $X$ is a nonresonant multibody final state. The only assumption is that the weak hadronic current is conserved. For the vector part of the current, corrections to this assumption are suppressed by $m_u - m_d$, while for the axial part they are suppressed by $m_u + m_d$. Perturbative electroweak corrections have been neglected in Eq. (4).

The CLEO collaboration has measured $v_X(m_X^2)$ for two-, three-, and four-pion final states [8,9]. The two-pion final states are quasi-two-body, being dominated by the $\rho$ resonance. Since the $\rho$ is narrow, ${\rm d}\Gamma(B \to D^{(*)}\pi\pi)/{\rm d}m_X^2$ is a steeply falling function of $m_X^2$ in the region we are interested in, and so is less useful for our purposes than the three- and four-pion final states. The three-pion final states receive a large $a_1$ contribution, however, the $a_1$ is rather broad and there is evidence for nonresonant contributions in $\tau$ decay to three pions. The four-pion final states are not dominated by any one resonance.

In terms of $v_X(m_X^2)$, the factorization prediction for the decay $B \to D^{(*)} X$ is

$$\frac{{\rm d}\Gamma(B \to D^{(*)} X)/{\rm d}m_X^2}{{\rm d}\Gamma(B \to D^{(*)} \ell\bar\nu)/{\rm d}m_X^2} = 3\pi \left( c_1(m_b) + \frac{c_2(m_b)}{3} \right)^2 v_X(m_X^2)\, (1 + \delta_{NF}) \,, \tag{5}$$

where $\delta_{\rm NF}$ denotes the nonfactorizable contributions, and in the denominator of the left-hand side $m_X$ denotes the dilepton invariant mass. In the large $N_c$ limit $\delta_{NF}$ is of order $1/N_c$, whereas in the perturbative QCD approach it is expected to contain both perturbative corrections and nonperturbative ones suppressed by powers of $m_X/E_X$, where $E_X = (m_B^2 + m_X^2 - m_{D^{(*)}}^2)/(2m_B)$ is the energy of the state $X$ in the $B$ rest frame. The ratio $m_X/E_X$ changes from 0.43 at $m_X = 1\,{\rm GeV}$ to 0.70 at $m_X = m_\tau$, so if there were order $m_X/E_X$ corrections to factorization then the accuracy of the predictions should change significantly over the accessible range of $m_X$.

In Fig. 1 we plot the $m_X^2$ distribution in $B \to D^* \pi^+ \pi^- \pi^- \pi^0$ decay, normalized to the $B \to D^* \ell \bar\nu$ rate, as predicted by Eq. (5). We used the CLEO fit to the shape of ${\rm d}\Gamma(B \to D^* \ell \bar\nu)/{\rm d}m_X^2$ [10] and the $\tau$ decay data [9]. This decay mode is interesting for testing factorization because it is not dominated by a single narrow resonance, and its branching fraction is large, almost 2% [11]. However, this mode has a significant disadvantage for testing factorization because of a potentially sizeable background from processes where one or more of the pions are emitted from the $(\bar c b)$ current. In this case, the $(\bar c b)$ current creates a charm state $C$ of the type $D^* + n\,\pi$ $(1 \le n \le 3)$, and the $(\bar d u)$ current creates the remaining

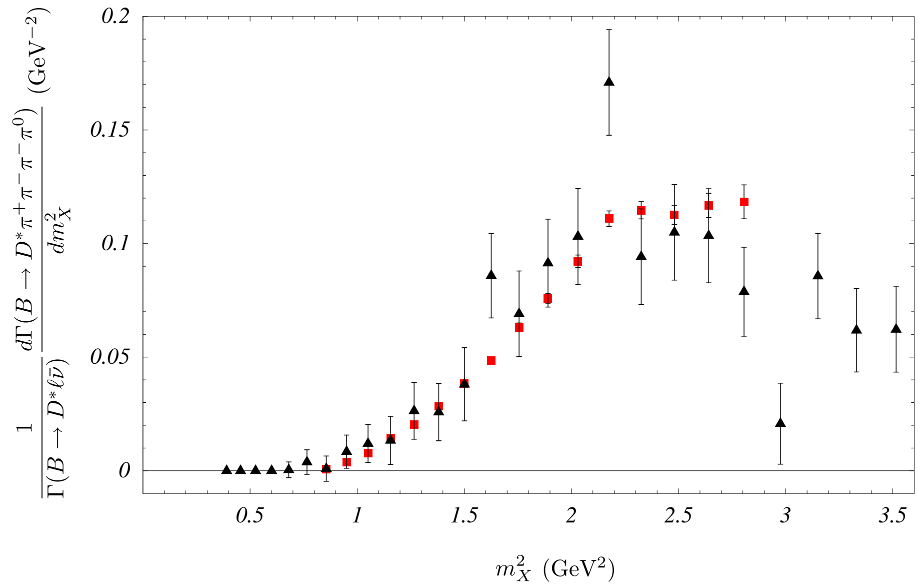


FIG. 1. $\mathrm{d}\Gamma(B \to D^*\pi^+\pi^-\pi^-\pi^0)/\mathrm{d}m_X^2$, where $m_X$ is the $\pi^+\pi^-\pi^-\pi^0$ invariant mass, normalized to the semileptonic width $\Gamma(B \to D^*\ell\bar\nu)$. The triangles are the $B$ decay data [11,12], and the squares are our predictions using the $\tau$ data [9]. There is an additional 9% uncertainty in the $B$ decay data from the overall normalization [12].

$4-n$ pions in the final state. In the formal limit when $m_b - m_c$ is large this background is parametrically suppressed, since the light hadrons from the $C$ decay combined with those from the $(\bar d u)$ current will have large invariant mass over most of the phase space. This can, however, be a sizable background for the physical $m_b$ and $m_c$ masses.

Because of the problem of backgrounds of this type, a more promising way to test the $m_X^2$ dependence of factorization is to focus on the subset of these decays which proceeds via $B \to D^*\omega\pi^-$. In this case, the backgrounds are easier to constrain, because the $(\bar d u)$ current must create the $\pi^-$ in the final state. Therefore, the $(\bar c b)$ current would have to create charmed states, $C_{(D^*\omega)}$, which can decay into $D^*\omega$. This background can be related using factorization (in the most reliable case of the current only making one pion) to the $B \to C_{(D^*\omega)}\ell\bar\nu$ decay rate. Assuming that $C_{(D^*\omega)}$ is a narrow resonance,

$$\Gamma(B \to C\pi) = \frac{3\pi^2 |V_{ud}|^2 f_\pi^2}{m_B\, m_C} \left(c_1(m_b) + \frac{c_2(m_b)}{3}\right)^2 \left(\frac{\mathrm{d}\Gamma(B \to C\ell\bar\nu)}{\mathrm{d}w}\right)_{w_{\max}}, \qquad (6)$$

where $w = v_B \cdot v_C$. Assuming that the $C_{(D^*\omega)}$ mass is its minimal value $m_C^{\min} = m_{D^*} + m_\omega$ and that semileptonic $B \to C_{(D^*\omega)}\ell\bar\nu$ rate depends on $w$ as $\sqrt{w^2-1}$, we find that $\Gamma(B \to C_{(D^*\omega)}\pi) \simeq 0.1 \times \Gamma(B \to C_{(D^*\omega)}\ell\bar\nu)$. Therefore, this background can be constrained by measuring the $B \to D^*\omega\ell\bar\nu$ decay rate. The measured $B \to D^*\omega\pi^-$ branching fraction is about 0.3% [11]. Thus, for example, a bound of $\mathcal{B}(B \to C_{(D^*\omega)}\ell\bar\nu) < 0.3\%$ would imply that the feed-down from higher mass $C_{(D^*\omega)}$ states to the $D^*\omega\pi^-$ final state is less than 10%. This is a very conservative estimate, since the $C_{(D^*\omega)}$ branching fraction to $D^*\omega$ is probably significantly less than unity. Moreover, the $B \to C_{(D^*\omega)}\ell\bar\nu$ branching fraction is probably much smaller than 0.3%; this is certainly the case in most models [13]. Therefore, we expect

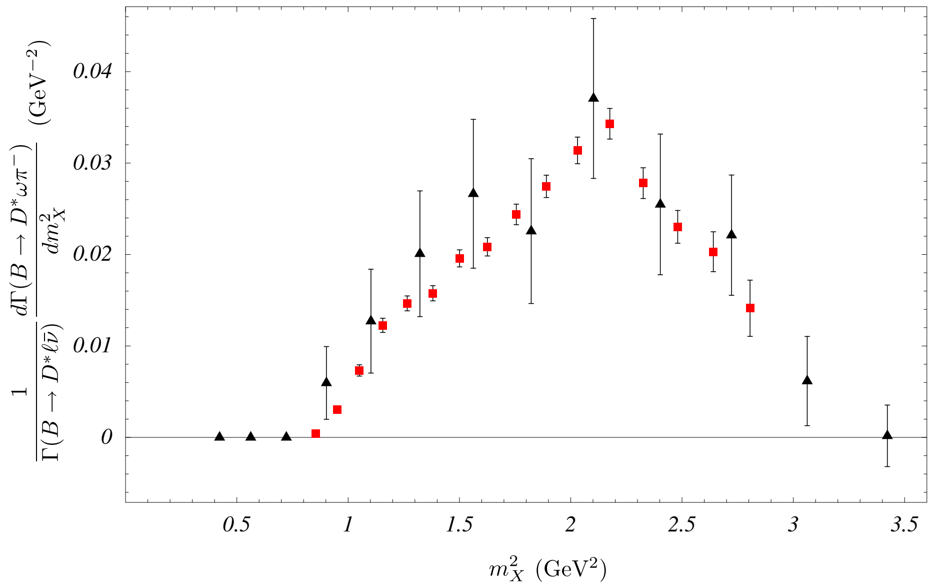


FIG. 2. $\mathrm{d}\Gamma(B \to D^*\omega\pi)/\mathrm{d}m_X^2$, where $m_X$ is the $\omega\pi^-$ invariant mass, normalized to the semileptonic width $\Gamma(B \to D^*\ell\bar{\nu})$. The triangles are the $B$ decay data [11], and the squares are our predictions using the $\tau$ data [9]. There is an additional 9% uncertainty in the $B$ decay data from the overall normalization [12].

that the $B \to D^*\omega\pi^-$ decay will only receive a very small background from higher mass charm states.

In Fig. 2 we plot the $m_X^2$ distribution in $B \to D^*\omega\pi^-$ decay, normalized to the $B \to D^*\ell\bar{\nu}$ rate, as predicted by Eq. (5) and the $\tau$ decay data [9]. This decay receives a large contribution from a $\rho'$ resonance with mass around 1.4−1.5 GeV [9,11]. However, the $\rho'$ is very broad, with a width estimated to be around $400 - 500$ MeV. Since we would like to make a precision test of factorization and learn about the size of corrections suppressed by powers of $m_X/E_X$ and $\alpha_s$, one needs tests that do not make assumptions based on resonance models or heavy quark symmetry [14], which may have corrections of similar size. This is best achieved by looking at the differential mass distribution, which should also teach us about how factorization breaks down (if it does at all) at higher masses.

Controlling this background in the $B \to D^*\pi^+\pi^-\pi^-\pi^0$ decay is more complicated than for $B \to D^*\omega\pi^-$, and may make it difficult to make a precise study of factorization in this channel. The contribution when $C$ contains two or three pions is expected to be small and can be constrained by bounding the semileptonic rates as in the case of $B \to D^*\omega\pi^-$. The contribution when one pion comes from the $(\bar{c}b)$ current and three pions from the $(\bar{d}u)$ current may be estimated from the measured $v_{3\pi}(m_X^2)$ and the $B \to D^*\pi\ell\bar{\nu}$ branching fraction. Using the ALEPH data [15] for $v_{3\pi}(m_X^2)$ measured in $\tau$ decay and $\mathcal{B}(B \to D^*\pi\ell\bar{\nu}) \sim 1\%$, it appears that this background may be significant. The size of this background can be tested experimentally by measuring the $B \to D^{*0}\pi^+\pi^+\pi^-\pi^-$ decay rate, since this decay only receives contributions from higher mass charm states which decay into a $D^{*0}$ plus one or three pions. Unless the $B \to D^{*0}\pi^+\pi^+\pi^-\pi^-$ rate is much smaller than the $B \to D^*\pi^+\pi^-\pi^-\pi^0$ rate, it will be difficult to make a precise test of factorization in $B \to D^*\pi^+\pi^-\pi^-\pi^0$.

Data on $e^+e^- \to$ hadrons can also be used to determine $v_X(m_X^2)$ and predict the mass spectrum in $B \to D^{(*)}X$ decay [16]. Currently the CMD–2 data on $e^+e^- \to 4\pi$ are available up to $m_X^2 = (1.4\,\mathrm{GeV})^2$ [17]; measurements of this cross section at higher energies would allow testing factorization at values of $m_X^2 > m_\tau^2$.

In summary, for a precision study of corrections to factorization in $B \to D^{(*)}X$ decays (where $X$ contains only light hadrons), it is best not to make assumptions based on heavy quark symmetry or resonant decomposition (especially for broad states). Rather, one should use the differential $m_X^2$ distribution with a fixed hadron content and compare with the predicted rate based on factorization using the spectral function extracted from $\tau$ decay. Over the accessible kinematic range the parameter $m_X/E_X$, which controls the accuracy of the predictions if the primary reason for factorization is perturbative QCD, changes from about 0.4 to 0.7, making $m_X$-dependent violations of factorization potentially observable. In this paper we concentrated on final states containing four pions, but this approach is general and can be applied to test factorization in decays to any light hadronic final state where backgrounds from the $(\bar{c}b)$ current producing some of the light hadrons can be controlled. Studying the available data in $B \to D^*\pi^+\pi^-\pi^-\pi^0$ and $B \to D^*\omega\pi^-$ decays, we found no evidence of factorization becoming a worse approximation at higher values of $m_X$. It will be very interesting to make a precision study, which should be possible at the $B$ factories. The decay mode $B \to D^*\omega\pi^-$ is particularly well suited for this purpose.

## ACKNOWLEDGMENTS

We thank Sheldon Stone, Jianchun Wang and Alan Weinstein for help with the CLEO data and discussions. Z.L. was supported in part by the Director, Office of Science, Office of High Energy and Nuclear Physics, Division of High Energy Physics, of the U.S. Department of Energy under Contract DE-AC03-76SF00098. M.L. was supported in part by the Natural Sciences and Engineering Research Council of Canada. M.B.W. was supported in part by the Department of Energy under Grant No. DE-FG03-92-ER40701.

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